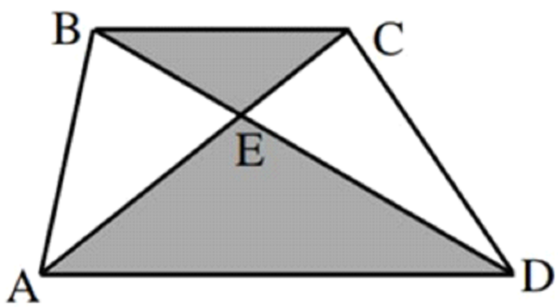
https://www.linkedin.com/feed/update/urn:li:activity:6712683386990845952

The diagonals of trapezoid $ABCD$ intersect at point E, forming triangle ADE with area 50 and triangle BCE with area 32.

What is the area of trapezoid $ABCD$?

Solution by Arkady Alt, San Jose, California, USA.



Let h_a and h_b be heights in triangles AED and BED from E to AD and BC, respectively.

Also let $a := AD, b := BC$, $F_a := [AED], F_b := [BED], F_l := [AEB] = [CED]$ ($[AEB] = [CED]$ because $[ABD] = [ACD]$ and $[AEB] = [ABD] - F_a = [ACD] - F_a = [CED]$).

Hence, $F_l = \dfrac{a(h_a + h_b)}{2} - \dfrac{ah_a}{2} = \dfrac{ah_b}{2}$. Similarly, by cosidering $\triangle ABC$ we can conclude

that $F_l = \dfrac{bh_a}{2}$ and, therefore, $F_l^2 = \dfrac{ah_b}{2} \cdot \dfrac{bh_a}{2} = F_aF_b \Leftrightarrow F_l = \sqrt{F_aF_b}$.

Thus, $[ABCD] = F_a + F_b + 2\sqrt{F_aF_b} = \left(\sqrt{F_a} + \sqrt{F_b}\right)^2$.

In particular for $F_a = 50, F_b = 32$ we obtain $[ABCD] = \left(\sqrt{50} + \sqrt{32}\right)^2 = 162$.